A cure for stuttering parity games

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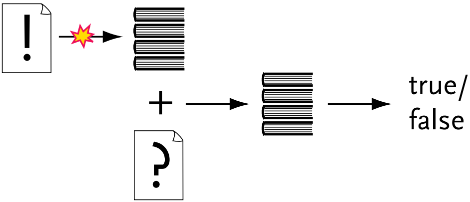
Bangalore, September 27, 2012

Technische Universiteit
Eindhoven
University of Technology

Where innovation starts

Context
Model checking

Traditional



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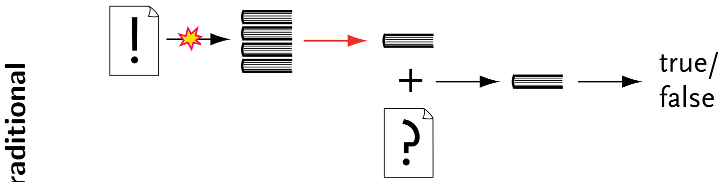

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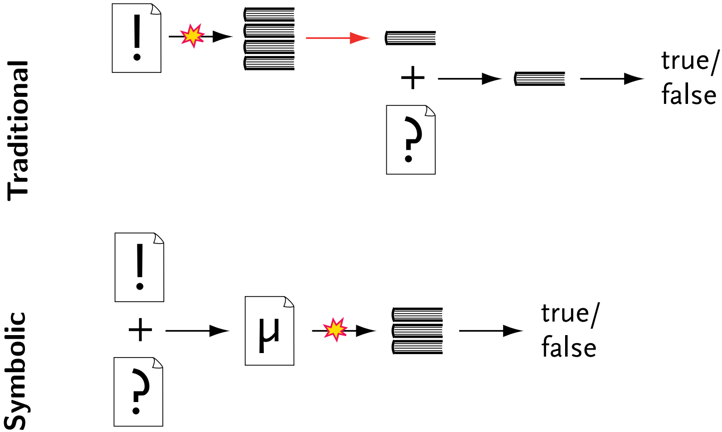

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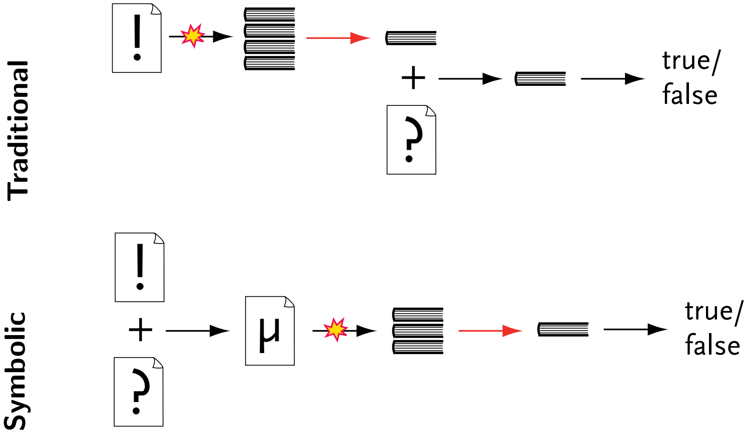

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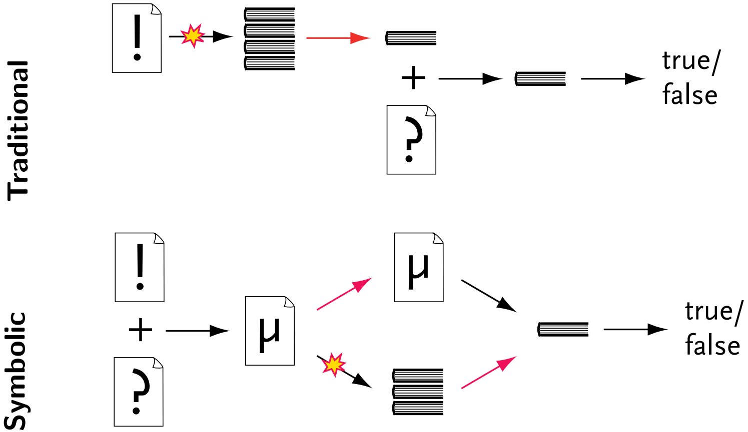
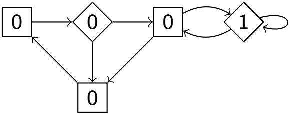


Parity games

V	A set of vertices.
$\rightarrow \subseteq V \times V$	An edge relation.
$\Omega : V \rightarrow \mathbb{N}$	A priority mapping.
$\mathcal{P} : V \rightarrow \{\Diamond, \Box\}$	A player mapping.



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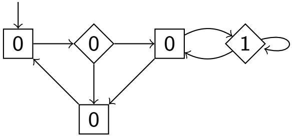

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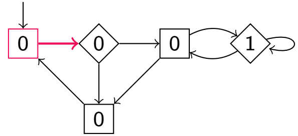

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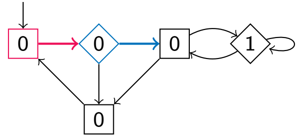

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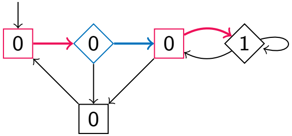

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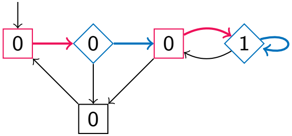

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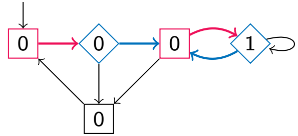

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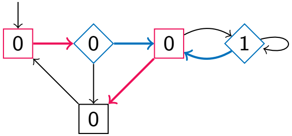

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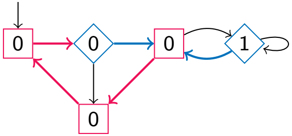

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Parity games
Goal

Decide winner equivalence: $\sim$

Parity games
Goal

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$NP \cap \text{co-}NP$

Parity games

Goal

some other

Decide ~~winner~~ equivalence:

P

Definition (Strong bisimilarity)

$\leftrightarrow\!\!\!\!\!\!{}_{-} \subseteq V \times V$ is the largest relation such that $v \leftrightarrow\!\!\!\!\!\!{}_{-}\; v'$ iff

- $\Omega(v) = \Omega(v')$ and $\mathcal{P}(v) = \mathcal{P}(v')$
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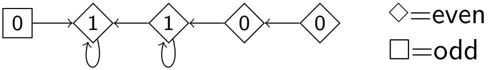

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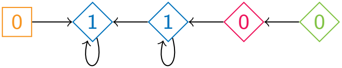

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Equivalences
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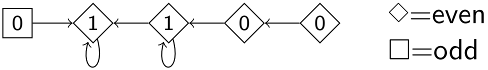

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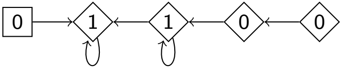

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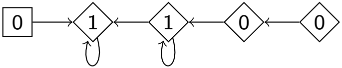

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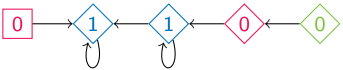

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Notation

$v \mapsto_{\simeq} \mathcal{C}$ means $\mathcal{C}$ is eventually reached by computation path through $\simeq$-related nodes.

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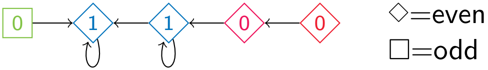

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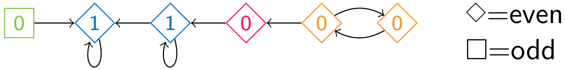

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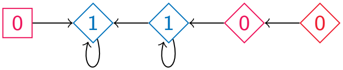

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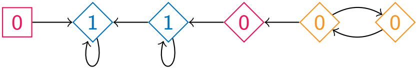

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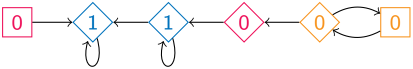

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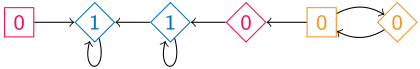

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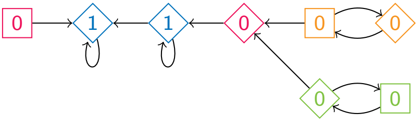

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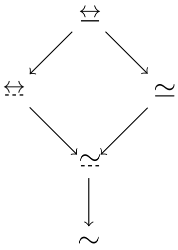
Equivalences



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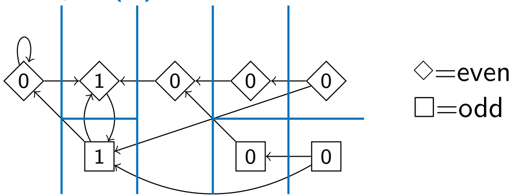

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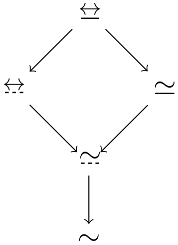

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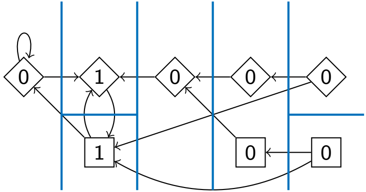

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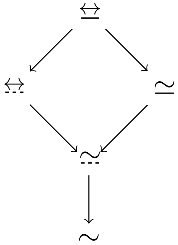

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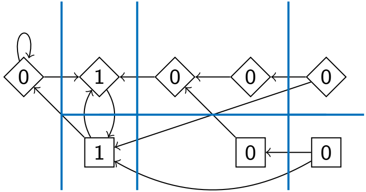

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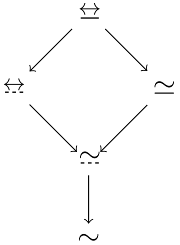
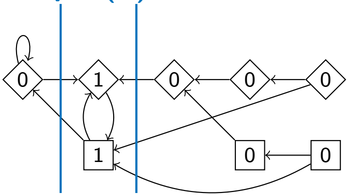


Example (≃)



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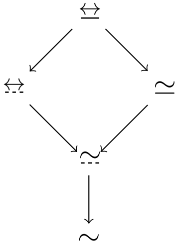

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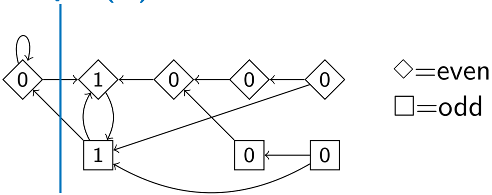


Equivalences
Complexity

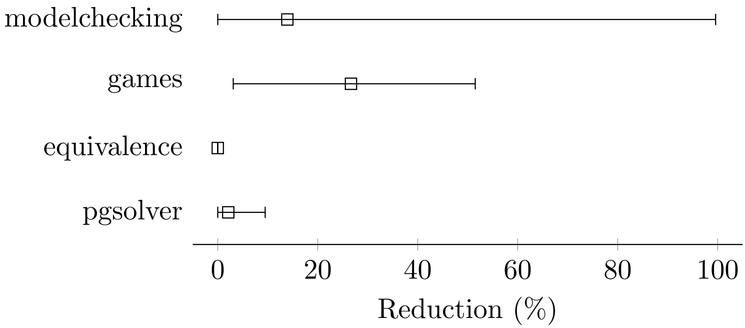
- $\simeq$ defines an equivalence relation.
- Can reduce a parity game, but by how much?
- Decidable in $O(|V|^2 \cdot |E|)$ time.
- Open problem: decide in $O(|V| \cdot |E|)$ time.

Test setup

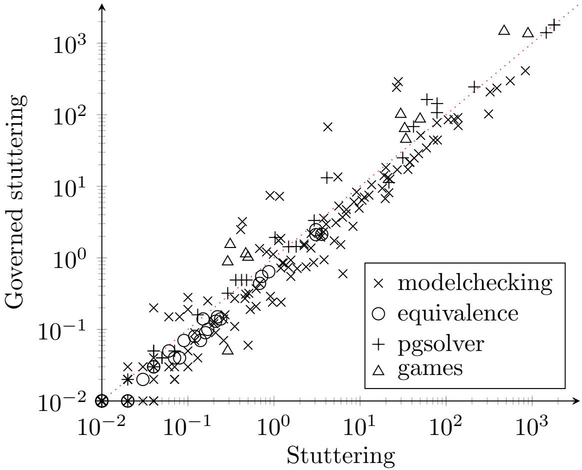
- 3 test sets ($>$300 instances).
  - Model checking problems.
  - Two player board games.
  - Equivalence checking problems.
  - PGSolver problems (also modelchecking).
- One $\simeq$-reduction, one $\simeq$-reduction (dashed underline).
  - Compare reduced sizes.
- Many solvers (PGSolver, own implementations).
  - Compare reduction + solving times.
  - Use fastest solving times.

Results

Size reduction (w.r.t. stuttering)



Results
Time reduction (w.r.t. stuttering)



Conclusion

Governed stuttering reduction mostly ...

- ... skims off the easy part of PG.
- ... does not relate much more than stuttering.
- ... does not solve games faster than stuttering.

Good size reduction of equivalences promising for symbolic setting.

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@jkeiren #ICTAC2012