Tailoring behavioural equivalences to parity games

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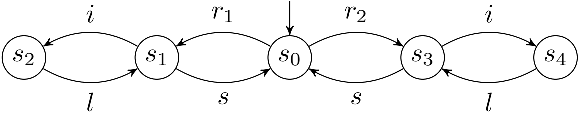
Where innovation starts

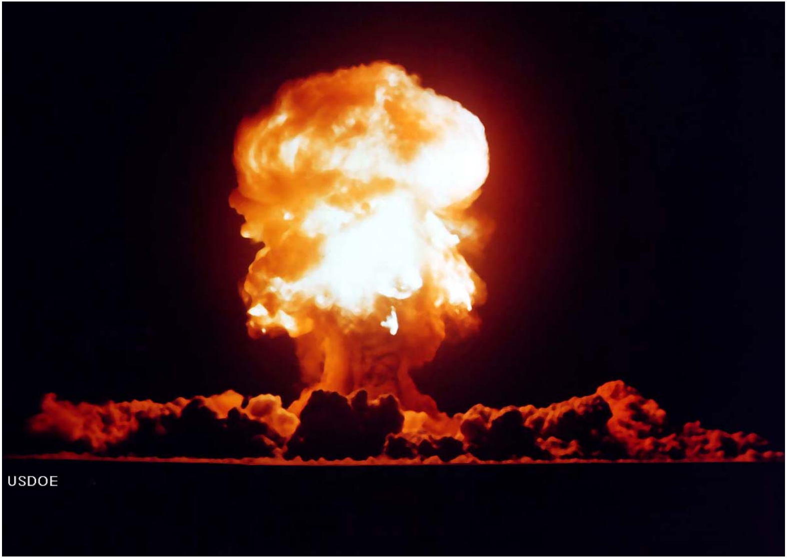
Model checking

- Automated
- Exhaustive

Transition system



$$L = \langle S, s_0, Act, \rightarrow \rangle$$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Bisimulation

Definition

Let $L = \langle S, Act, \rightarrow \rangle$ be a labelled transition system. A symmetric relation $R \subseteq S \times S$ is a strong bisimulation if for all $(s, s') \in R$

$$\forall a \in Act, t \in S : s \xrightarrow{a} t \implies \exists t' \in S : s' \xrightarrow{a} t' \wedge (t, t') \in R$$

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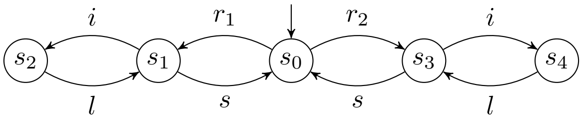

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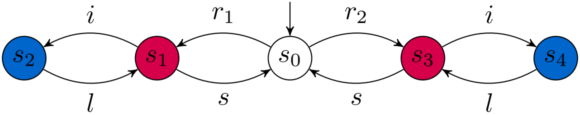

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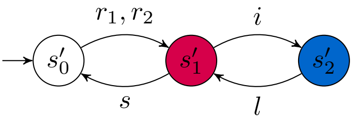
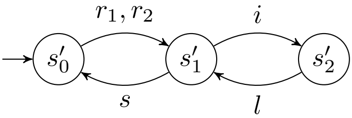


Parity game



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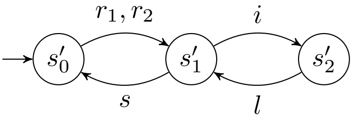
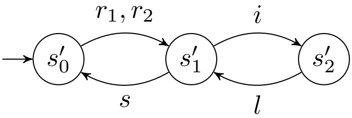
Parity game



$$+$$

$$\nu X.((\mu Y.\langle r_1\rangle X \vee \langle\overline{r_1}\rangle Y) \wedge (\mu Z.\langle r_2\rangle X \vee \langle\overline{r_2}\rangle Z))$$

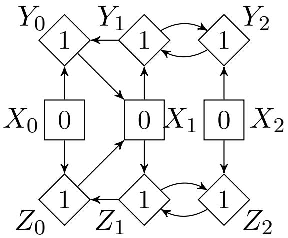
Parity game



$$+$$

$$\nu X.((\mu Y.\langle r_1\rangle X \vee \langle \overline{r_1}\rangle Y) \wedge (\mu Z.\langle r_2\rangle X \vee \langle \overline{r_2}\rangle Z))$$

$$\Downarrow$$



Bisimulation

Definition

Let $\mathcal{G} = (V, \to, V_{Even}, V_{Odd}, \Omega)$ be a parity game. A symmetric relation $R \subseteq S \times S$ is a strong bisimulation if for all $(v, v') \in R$

- $v \in V_{Even} \Leftrightarrow v' \in V_{Even}$
- $\Omega(v) = \Omega(v')$
- $\forall w \in V : v \to w \implies \exists w' \in V : v' \to w' \wedge (w, w') \in R$

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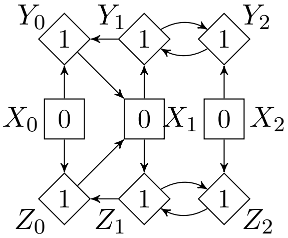
Example



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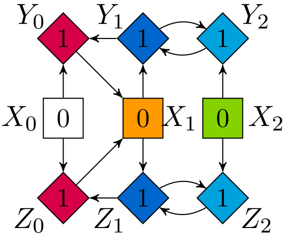

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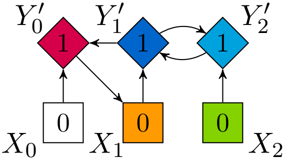
Example



Experiments (1)

If sending some message is enabled infinitely often, then it is sent infinitely often

Model	Original	Reduced
CABP (16)	1,732,627	1,238
SWP (4)	8,957,959	25,354

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Experiments (2)

It is always possible for the system to get to a state in which pressing the DOWN button of any lift will yield the appropriate response

Model	Original	Reduced
Lift (3) - init	17,203	729
Lift (4) - init	189,801	5,054
Lift (3) - final	4,648	111
Lift (4) - final	23,437	171

Experiments (3)

If one of the lifts moves, all other lifts should not move in the opposite direction. (variation)

Model	Original	Reduced
Lift (3) - init	318,213	2,475
Lift (4) - init	16,416,476	14,639
Lift (3) - final	13,403	729
Lift (4) - final	200,505	2,190

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Experiments (n)

Branching bisimilarity of two specifications

Model 1	Model 2	Original	Reduced
ABP (16)	CABP (16)	4,054,706	21,329
ABP (2)	SWP (2)	1,864,138	434,820

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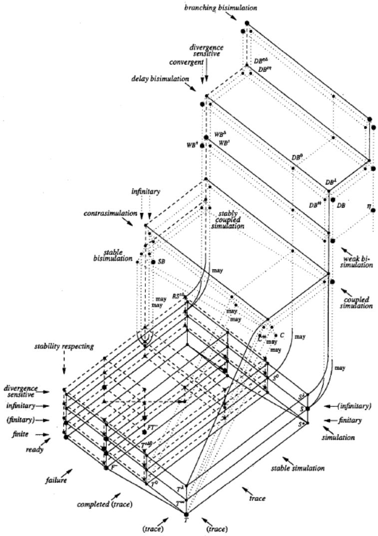

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Weaker equivalences

- Relate vertices of different players
- Compress sequences of similar states
- Combine these approaches

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Future work & conclusions

- Use notions to simplify PBESes
- Other relations, e.g. simulation
- Algorithm for weakest proposed equivalence
- Identify classes solved by equivalences