A cure for stuttering parity games

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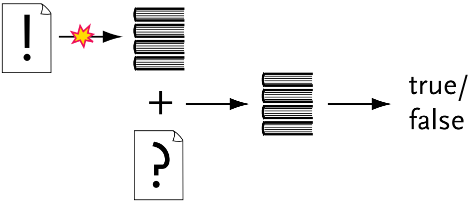
VU Amsterdam, November 2, 2012

Technische Universiteit
Eindhoven
University of Technology

Where innovation starts

Context
Model checking

Traditional



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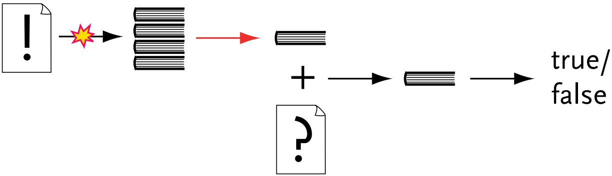

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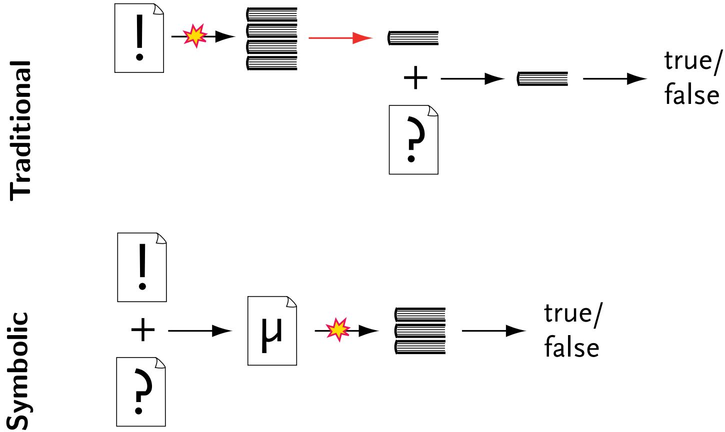

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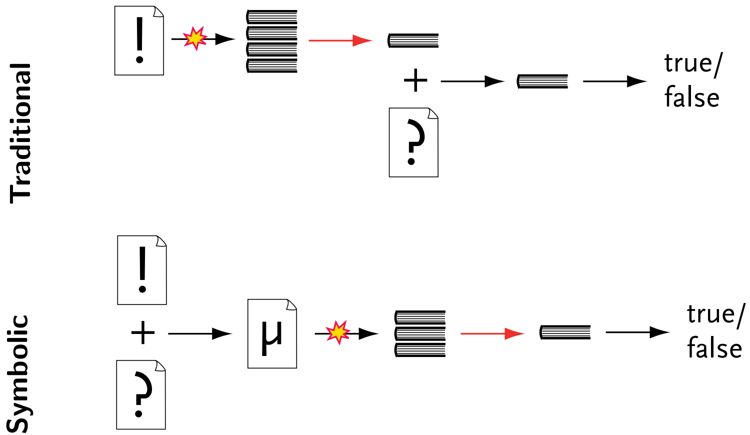

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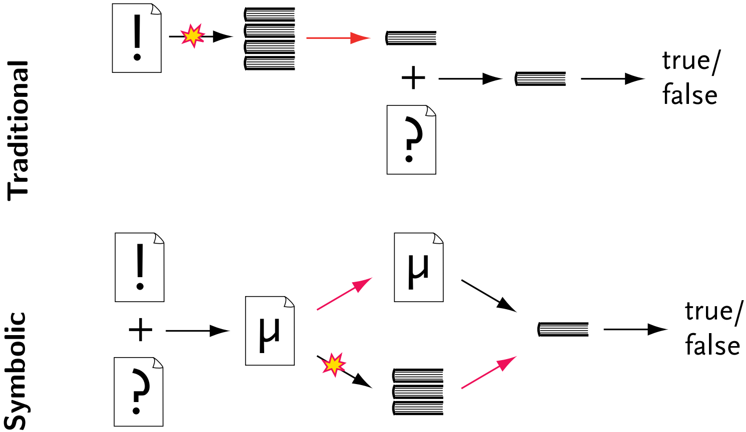
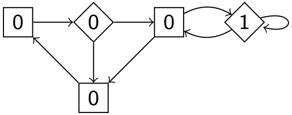


Parity games

V	A set of vertices.
$\rightarrow \subseteq V \times V$	An edge relation.
$\Omega : V \rightarrow \mathbb{N}$	A priority mapping.
$\mathcal{P} : V \rightarrow \{\Diamond, \Box\}$	A player mapping.



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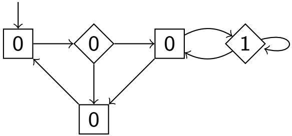

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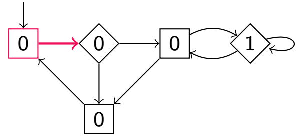

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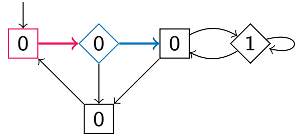

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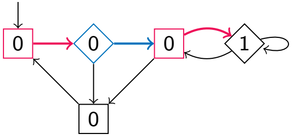

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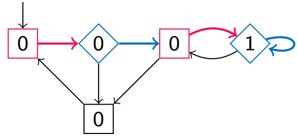

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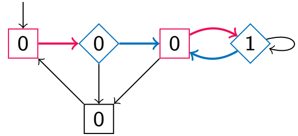

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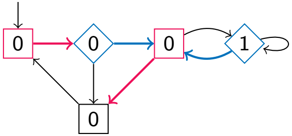

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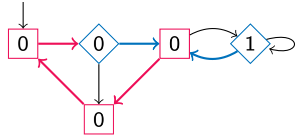

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Parity games
Goal

Decide winner equivalence: $\sim$

Parity games

Goal

Decide winner equivalence: $\sim$

$NP \cap \text{co-}NP$

Parity games
Goal

some other

Decide ~~winner~~ equivalence:

P

Definition (Strong bisimilarity)

$\leftrightarroweq \subseteq V \times V$ is the largest relation such that $v \leftrightarroweq v'$ iff

- $L(v) = L(v')$
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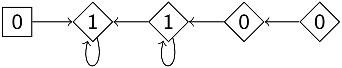

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Equivalences
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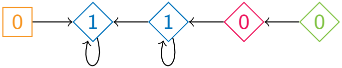

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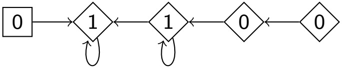

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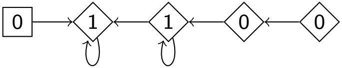

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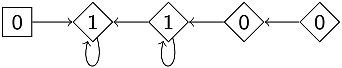

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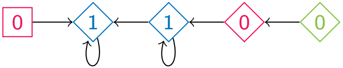

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Notation

$v \mapsto_{\simeq} \mathcal{C}$ means $\mathcal{C}$ is eventually reached by computation path through $\simeq$-related nodes.

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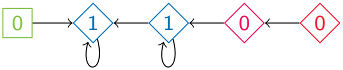

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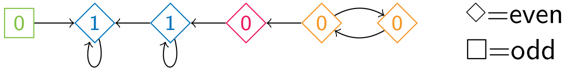

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$v \, {}_{\mathcal{P}(v)}{\mapsto}_{\simeq} \mathcal{C}$ means $\mathcal{P}(v)$ has strategy to force game to $\mathcal{C}$ through $\simeq$-related nodes.

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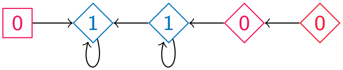

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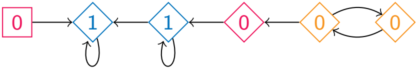

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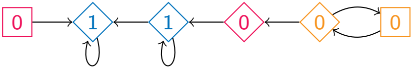

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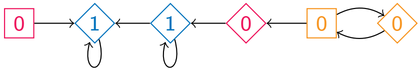

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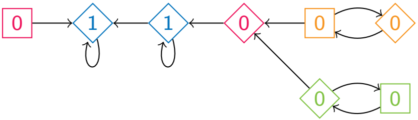

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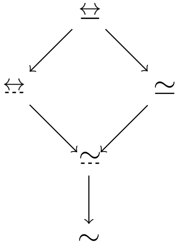

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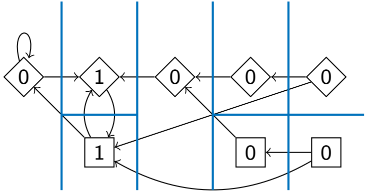

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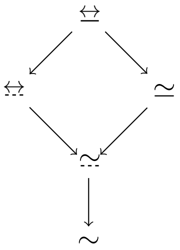

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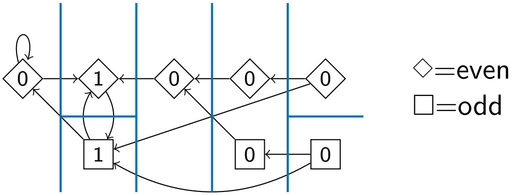

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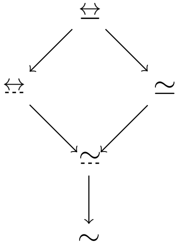

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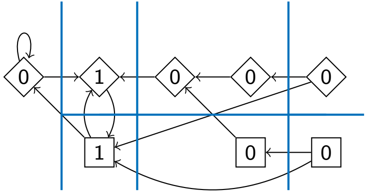

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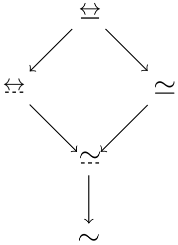

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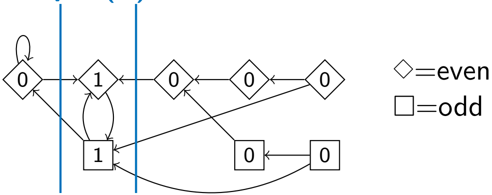
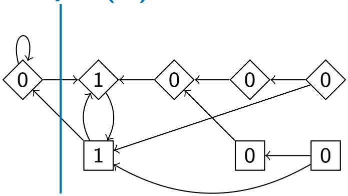


Equivalences

$$\begin{tikzcd}[column sep=small] & \underline{\leftrightarrow} \arrow[dl] \arrow[dr] & \\ \underline{\underline{\leftrightarrow}} \arrow[dr] & & \underline{\simeq} \arrow[dl] \\ & \underline{\underline{\simeq}} \arrow[d] & \\ & \sim & \end{tikzcd}$$

Example ($\sim$)



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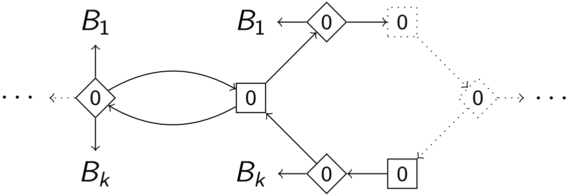
Equivalences
Complexity

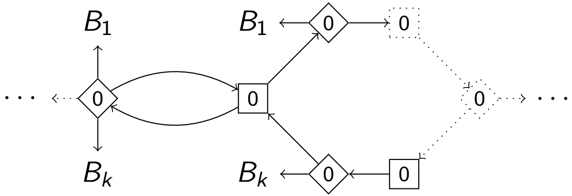
- $\simeq$ defines an equivalence relation.
- Can reduce a parity game, but by how much?
- Decidable in $O(|V|^2 \cdot |E|)$ time.
- Open problem: decide in $O(|V| \cdot |E|)$ time.

Problem

Let P be partition, $C \in P$ a block, s.t. C is a SCC, with $v \;{}_i{\mapsto}_P$, and $v \;{}_{\neg i}{\not\mapsto}_P$ for all $v \in C$. Determine, in $\mathcal{O}(m)$ time, whether there exist $v, v' \in C$ and a block $B \in P$ such that $v \;{}_i{\mapsto}_P\, B$ and $v' \;{}_i{\not\mapsto}_P\, B$.

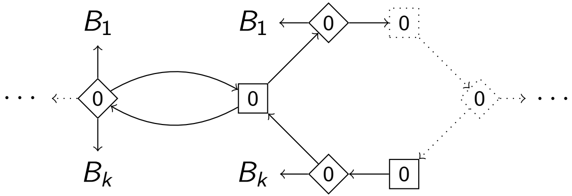


Complexity
Open problem (ctd.)



We need $\Omega(km_C)$ time to conclude that there are no splitters

Complexity
Open problem (ctd.)



We need $\Omega(km_C)$ time to conclude that there are no splitters

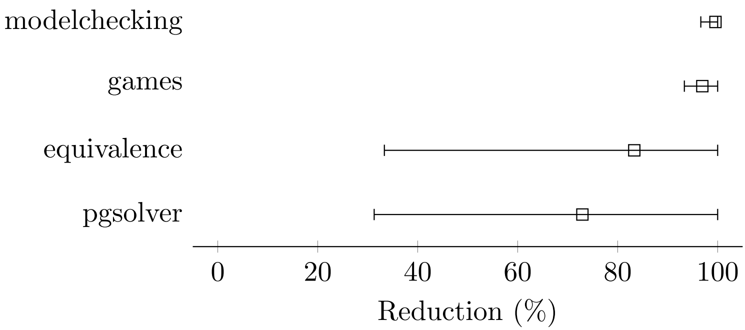
Conjecture

If we can find splitter in $\mathcal{O}(m_C)$ time, the largest governed stuttering can be computed in $\mathcal{O}(|V| \cdot |E|)$ time

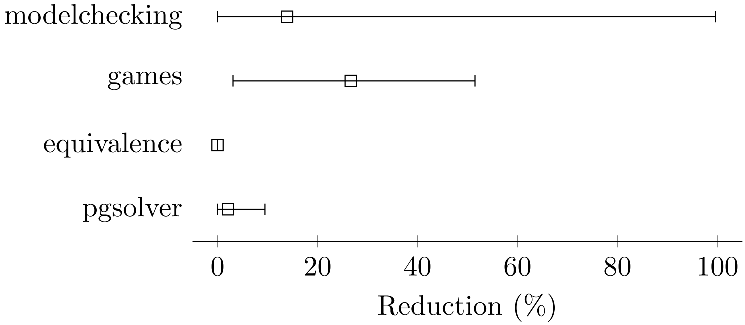
Test setup

- 3 test sets (>300 instances).
  - Model checking problems.
  - Two player board games.
  - Equivalence checking problems.
  - PGSolver problems (also modelchecking).
- One $\simeq$-reduction, one $\simeq$-reduction.
  - Compare reduced sizes.
- Many solvers (PGSolver, own implementations).
  - Compare reduction + solving times.
  - Use fastest solving times.

Results
Size reduction (w.r.t. original)

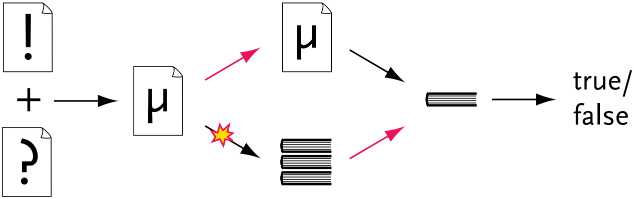


Results
Size reduction (w.r.t. stuttering)

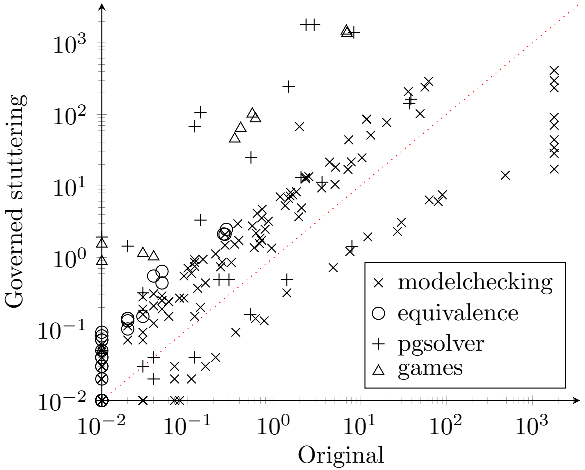


Results
Size reduction

- Vast reduction w.r.t. original.
- Reduces graph much more than bisimulation.
- Reduction similar to stuttering equivalence.
- Promising for symbolic setting.



Results
Time reduction (w.r.t. original)

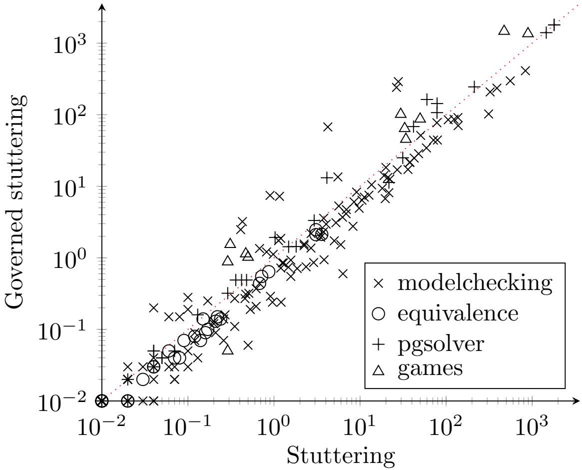


Results
Solving times

- Slows down on average.
- Highly dependent on
  - available solver implementations, and
  - implementation of reduction.
- Some timeout cases became solvable.

Results

Time reduction (w.r.t. stuttering)



Conclusion

Governed stuttering reduction mostly ...

- ... skims off the easy part of PG.
- ... does not relate much more than stuttering.
- ... does not solve games faster than stuttering.

Good size reduction of equivalences promising for symbolic setting.