Benchmarking Parity Games

FSEN 2014

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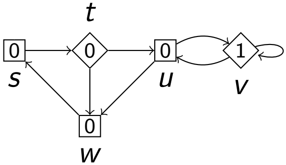
Where are parity games used?

- Model Checking
- Equivalence Checking
- Satisfiability/Validity of modal logic
- Synthesis

Parity Games

V	A set of vertices.
$\rightarrow \subseteq V \times V$	An edge relation.
$\Omega : V \rightarrow \mathbb{N}$	A priority mapping.
$\Diamond, \Box$	Two players (even, odd).
$(V_{\Diamond}, V_{\Box})$	A partition of V.



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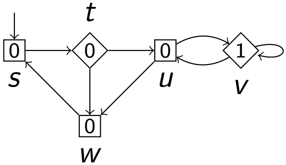

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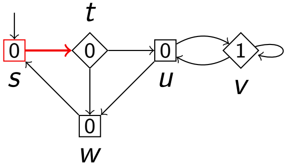

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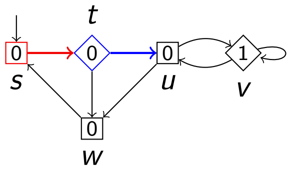

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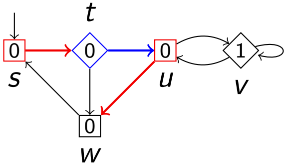

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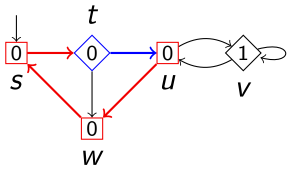

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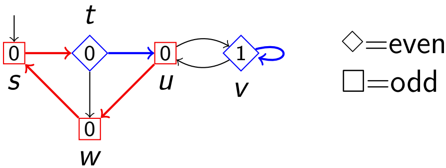

- Winner?

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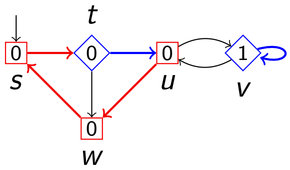

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- Winner?
- Optimal strategies?

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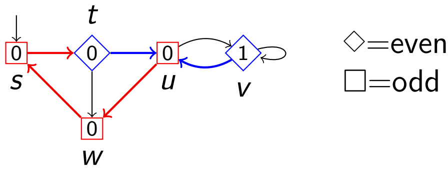

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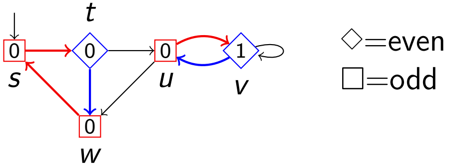

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- Optimal strategies?

Winning Parity Games

Memoryless determinacy

- Partition $(W_{\Diamond}, W_{\Box})$ of V
- Player $\bigcirc$ has memoryless winning strategy from $W_{\bigcirc}$, for $\bigcirc \in \{\Diamond, \Box\}$

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Solving Parity Games

Solving a parity game:

- Determine partition $(W_{\Diamond}, W_{\Box})$

Complexity:

- Problem is in $NP \cap \text{co-}NP$
- Is it in P?

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Solving Parity Games

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- Determine partition $(W_{\Diamond}, W_{\Box})$

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Why benchmark?

Complexity $+$ applications $\Rightarrow$ active research

Algorithms for:

- solving
- simplifying
- reducing

parity games

How to compare new algorithms to existing ones?

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Existing practice

- Only theoretical analysis (big-O)
- Class of games that meets upper bound
- Random games
- (Very) small set of games

Results from different papers not comparable

Requirements on Benchmarks

- Cover broad range of games:
  - Different problems
  - Different structural properties
- Games from the literature

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Contributions

- Set of parity games
- List of structural properties
- Analysis of games w.r.t. these properties

Set of parity games

- Model checking:
  - Communication protocols (C)ABP, BRP, SWP
  - Cache coherence protocol
  - Two-player board games
  - Industrial IEEE-1394 link-layer, truck lift
  - Elevator, Hanoi towers
- Equivalence checking: strong-, weak-, branching bisimulation of communication protocols
- Validity/satisfiability of LTL, CTL, CTL*, PDL and μ-calculus (using MLSolver)
- Random games (using PGSolver)
- Hard cases (using PGSolver)

Structural properties

- Some properties known to affect complexity of solving:
  - Number of vertices and edges ("size")
  - Number of priorities
  - Width measures (tree-width, DAG-width, etc.)
  - SCCs
- New: alternation depth (inspired by modal equation systems)
- And some more...

Alternation depth

- Describe complexity more accurately
- Similar to ideas in [Emerson & Lee 1986] for μ-calculus

Three steps (let $\mathcal{C} \in \mathsf{sccs}(G)$)

1. Nesting depth of v in $\mathcal{C}$ is #alternations between even and odd priorities on paths of descending priorities in $\mathcal{C}$
2. Nesting depth of $\mathcal{C}$ is $\max\{nestingdepth(v) \mid v \in \mathcal{C}\}$
3. Alternation depth of of a parity game is the maximal nesting depth of its SCCs

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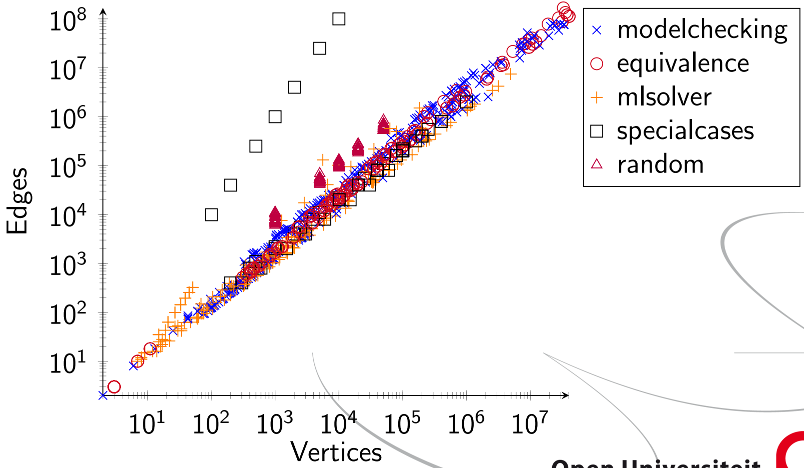
Analysis of games w.r.t. structural properties

Vertices vs. edges



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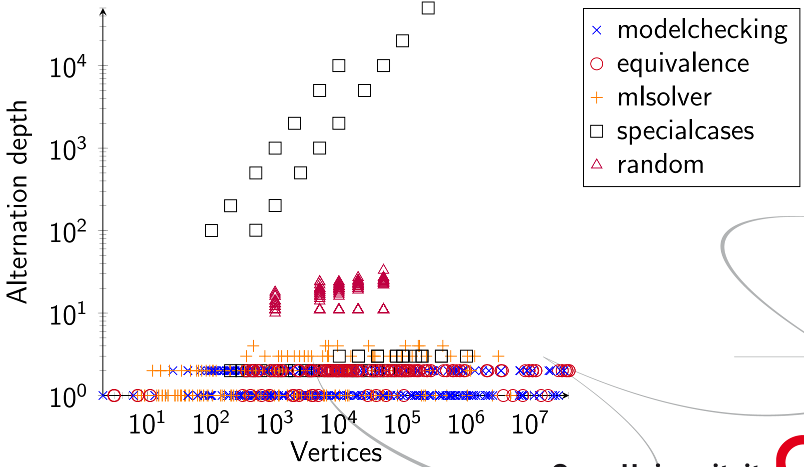
Analysis of games w.r.t. structural properties

Alternation depth



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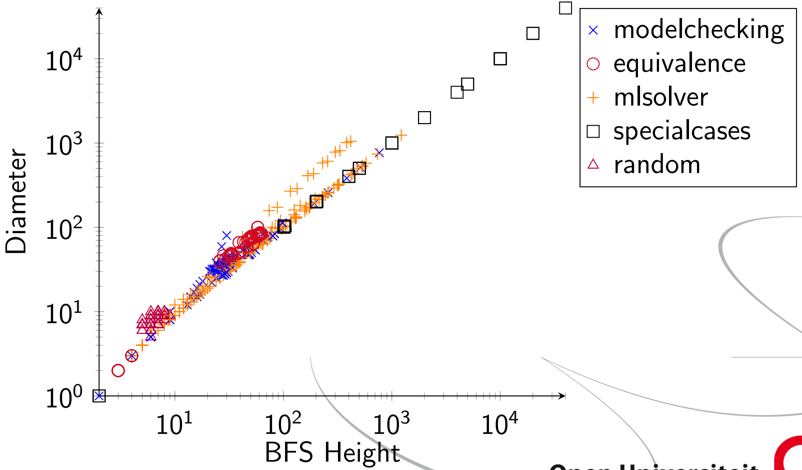
Analysis of games w.r.t. structural properties

Diameter



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Applications

- Used to assess parity game reductions in [Cranen, K & Willemse 2011,2012]
- Subset of generation process used for benchmarks in [K, Wesselink & Willemse, 2014]
- Confirmed observation from [Friedmann & Lange 2009]: recursive algorithm beats sophisticated algorithms (unpublished)

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Summary

I presented:

- A set of parity games
- Structural properties of parity games
- An analysis of the games w.r.t. these properties

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Open issues

- Use structural properties to optimise/design algorithms
- Perform large-scale comparison of different algorithms
- Extend set of games with other encodings/more examples
- Design algorithms for computing more complex structural properties

Please contribute your own games!

jeroenkeiren.nl
github.com/jkeiren/paritygame-generator

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